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TRANSFER MAP APPROACH TO THE BEAM-BEAM INTERACTION

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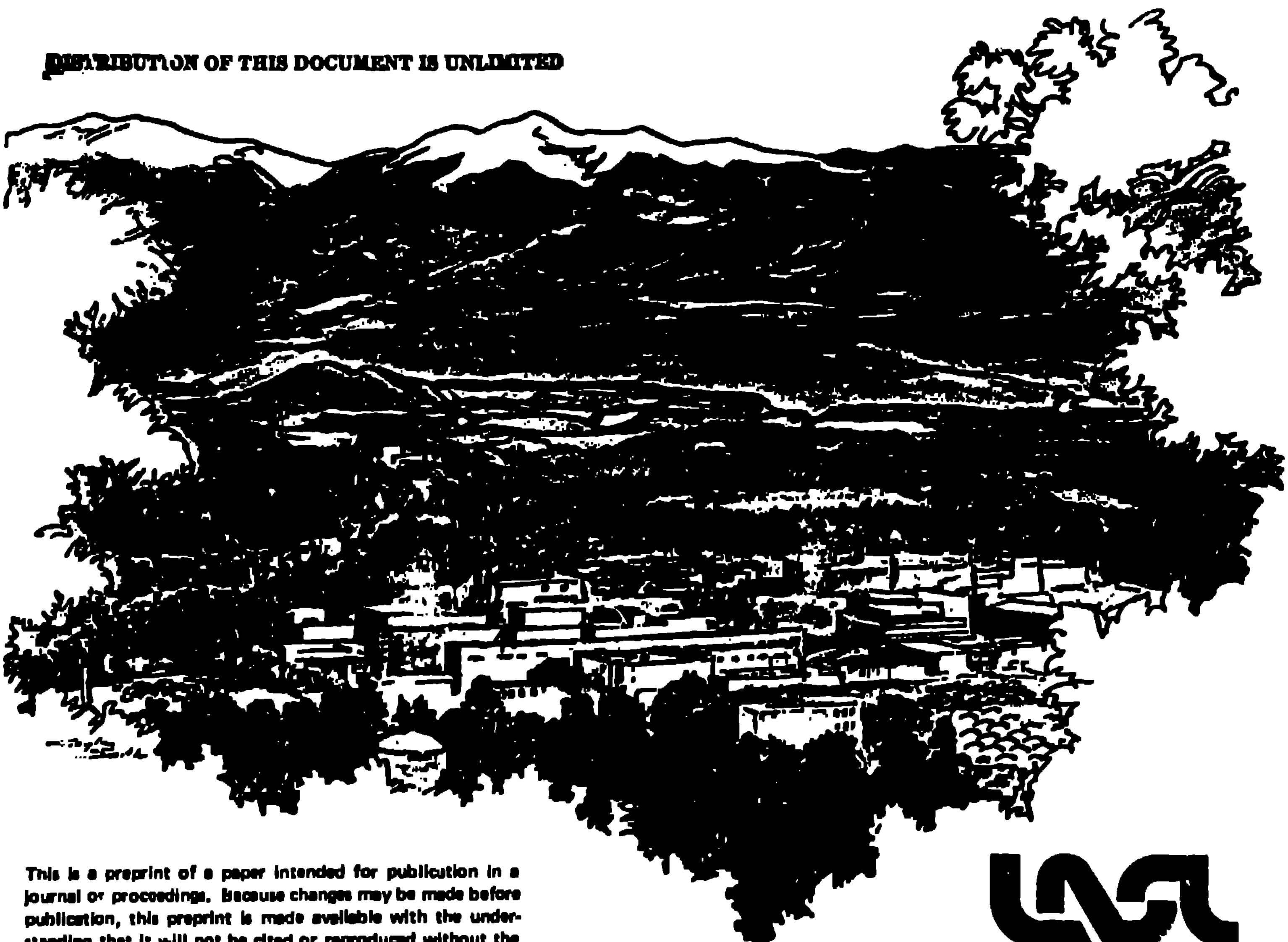

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Abstract

A study is made of a model for the beam-beam interaction in ISABELLE using numerical methods and the recently developed method of Transfer Maps. It is found that analytical transfer map calculations account qualitatively for all the features of the model observed numerically, and show promise of giving quantitative agreement as well. They may also provide a kind of "magnifying glass" for examining numerical results in fine detail to ascertain the presence of small scale stochastic motion that might lead to eventual particle loss. Preliminary evidence is presented to the effect that within the model employed, the beam-beam interaction at its contemplated strengths should not lead to particle loss in ISABELLE.

1. Introduction

The purpose of this paper is to explore the model of Herrera, Month, and Peierls[1] for the ISABELLE beam-beam interaction with the aid of the recently developed method of Transfer Maps[2] and its associated Lie algebraic techniques.[3] The model employed for the beam-beam interaction is "weak-strong":[1,4] One beam, the strong beam, is taken to be fixed, and the other beam, the weak beam, is treated as a collection of particles that are affected by their passage through the strong beam but not by each other. The strong beam is assumed to be an unbunched ribbon in the horizontal plane whose vertical charge distribution is well described by a Gaussian shape. The weak beam also lies in the same horizontal plane and crosses the strong beam at a fixed angle. Only vertical deflections of the weak beam by the strong beam are taken into account.

In the strong beam-weak beam limit, the net motion of a particle in the weak beam can be viewed as the continual repetition of two sequential motions: passage through the storage ring followed by passage through the strong beam. See Fig. 1. The equations of motion for each of these two passages (through the ring and through the strong beam) are derivable from Hamiltonians, and therefore each passage is described by a symplectic (Poisson bracket preserving) transfer map.[2,3]

By design, the passage through a storage ring is well described by a linear map. Upon restricting attention only to vertical motion and making a suitable choice of coordinates, the "ring" transfer map can be written as

$$
\begin{aligned}
q' &= q \cos(2\pi w) + p \sin(2\pi w) \\
p' &= - q \sin(2\pi w) + p \cos(2\pi w) \ .
\end{aligned}
\tag{1}
$$

Here q is proportional to the vertical coordinate of a particle in the weak beam, p is a suitably chosen canonically conjugate momentum, and w modulo an integer is the tune of the storage ring.[1] The unprimed variables q,p specify the particle orbit just as it enters the ring, and the primed variables q',p' describe the orbit upon exit.

The effect of passage through the strong beam is more complicated. To find the "beam" transfer map exactly, it is necessary to integrate the nonlinear equations of motion for a particle passing through the strong beam. However, a good approximation to this map is given by assuming that the particle suffers a vertical momentum change depending only upon its initial vertical position, and that the vertical position itself remains unaffected:

$$\begin{aligned} q'' &= q' \\ p'' &= p' + u(q') \ . \end{aligned} \tag{2}$$

This impulse approximation becomes exact in the limit that the interaction region becomes a point and/or the transit time through the region approaches zero. In any case, the beam mapping (2) is symplectic, and therefore its use will produce no qualitative error.

The function u is proportional to the electrostatic force exerted by the strong beam. In the coordinates and Gaussian model employed, u is given by the relation[1]

$$u(q) = 4\pi D/\sqrt{3} \int_0^{q\sqrt{3}} dt \; e^{-t^2} \ . \tag{3}$$

Here D is the beam-beam strength parameter that typically[4] has values ranging from 10^{-3} to 10^{-2}. It is normalized in such a way that the beam-beam interaction depresses the tune for infinitesimal betatron oscillations by an amount D when D is small.

There is one last caveat to be made. According to the current design, ISABELLE will actually have 6 collision regions separated by 6 identical lattice sections. Thus, in reality, the maps 1 and 2 must be iterated 6 times to simulate the effect of one complete turn. Correspondingly, equation (1) is the transfer map for one lattice section and w modulo an integer is actually 1/6 of the total machine tune.

In this paper the effect of repeated iteration of the maps (1) and (2) are studied using Transfer Map methods and results are compared with numerical calculations.

2. Transfer Map Results

To treat q and p on an equal footing, it is notationally convenient to introduce variables z_1 and z_2 by the relations

$$\begin{aligned} z_1 &= q \\ z_2 &= p\ . \end{aligned} \tag{4}$$

Employing this notation, let f(z) be any function of the phase-space variables z. With each such function f there is an associated Lie operator F. This operator acts on functions and is defined by the rule

$$Fg = [f,g]\ . \tag{5}$$

Here g is any function of the phase-space variables, and the square bracket [,] denotes the Poisson bracket operation familiar from classical mechanics.

Next, consider the object exp(F), called a Lie transformation, defined by the exponential series

$$\exp(F) = I + F + F^2/2! + F^3/3! + \ldots . \tag{6}$$

More explicitly, the action of exp(F) on an arbitrary function g is given by the expression

$$\exp(F)g = g + [f,g] + [f,[f,g]]/2! + \ldots . \tag{7}$$

Now consider the operator $\exp(F_2)$ where F_2 is the Lie operator associated with the quadratic polynomial

$$f_2 = -\pi w(z_1^2 + z_2^2) \; . \tag{8}$$

It is easily verified that

$$\begin{aligned} F_2\, z_1 &= [f_2,z_1] = 2\pi w z_2 \\ F_2\, z_2 &= [f_2,z_2] = -2\pi w z_1 \; . \end{aligned} \tag{9}$$

Consequently, use of (7) and (9) gives the relation

$$\begin{aligned} \exp(F_2)\, z_1 &= z_1 + z_2(2\pi w) - z_1(2\pi w)^2/2! \\ &\quad - z_2(2\pi w)^3/3! + \ldots \\ &= z_1 \cos(2\pi w) + z_2 \sin(2\pi w) \; . \end{aligned} \tag{10a}$$

Similarly, it can be checked that

$$\exp(F_2)\, z_2 = -z_1 \sin(2\pi w) + z_2 \cos(2\pi w) \; . \tag{10b}$$

Therefore the ring transfer map (1) can be written in the compact form

$$z' = \exp(F_2)\ z \ . \qquad (11)$$

A similar Lie transformation representation can be found for the beam transfer map (2). Let $f_b(z)$ be the function defined by the relation

$$f_b(z) = \int_0^{z_1} u(q)\ dq \ . \qquad (12)$$

The Poisson bracket relations analogous to (9) are

$$F_b\ z_1 = [f_b,\ z_1] = 0 \qquad (13)$$

$$F_b\ z_2 = [f_b,\ z_2] = \partial f_b/\partial z_1 = u(z_1)$$

$$F_b^2\ z_2 = [f_b,\ [f_b,\ z_2]] = [f_b,\ u(z_1)] = 0,\ \text{etc.}$$

Consequently the infinite sum (7) is trivial to evaluate in this case because it terminates. One finds the result

$$\exp(F_b)\ z_1 = z_1 \qquad (14)$$

$$\exp(F_b)\ z_2 = z_2 + u(z_1) \quad .$$

Therefore the beam transfer map (2) can be written in the form

$$z'' = \exp(F_b)\ z' \ . \qquad (15)$$

Combining the two results (11) and (15), one finds that the net transfer map $\mathcal{M}$ for passage through the ring followed by passage through the strong beam is given by the product

$$\mathcal{M} = \exp(F_2) \exp(F_b) \ . \tag{16}$$

The observant reader may be worried about the order in which the two factors appear in (16). It can be verified that the above order indeed is correct because Lie transformations have the property

$$\exp(F_2)\ g(z) = g\ \exp(F_2)\ z \ = g(z') \tag{17}$$

for any function $g(z)$.[3]

The problem at hand is to evaluate $\mathcal{M}^n$ for large n in order to compute the effect of many passages through the ring and the strong beam. The computation of $\mathcal{M}^n$ would be easy if a Lie operator H could be found such that $\mathcal{M}$ could be re-expressed in the form exp(H), for then $\mathcal{M}^n$ would be simply given by exp(nH). The determination of such an H is a standard problem in the theory of Lie algebras that is solved by using the Campbell-Baker-Hausdorff formula.[3] This formula gives H in terms of F_2 and F_b, and their multiple commutators. In addition, there is an analogous formula that gives the function h associated with H in terms of f_2 and f_b, and their multiple Poisson brackets. It also can be shown that the computation of exp(nH) is equivalent to the integration of a "trajectory" in "z space" for n units of "time" using -h as an "effective" Hamiltonian. Consequently, the function h(z) is formally invariant under the map. This means that the function h(z) generalizes the Courant-Snyder invariant to the case of nonlinear motion.

For the problem under consideration, h is given by the formal operator formula,

$$h = f_2 + F_2 \left[1 - \exp(-F_2)\right]^{-1} f_b + \dots . \tag{18}$$

The terms not shown in the series involve Poisson bruckets with more than one f_b, and therefore are quadratic and higher order in the beam-beam strength parameter. Consequently, as it stands, equation (18) is correct through first order in the beam-beam strength.

The computation of the effect of the operator F_2 and functions of F_2, such as occur in (18), is facilitated by the introduction of "polar" coordinates in phase space and the use of Fourier series. This can be achieved in a canonical way by using action angle variables a, ϕ defined by the relations

$$\begin{aligned} q = z_1 &= (2a)^{1/2} \sin \phi \\ p = z_2 &= (2a)^{1/2} \cos \phi . \end{aligned} \tag{19}$$

It is evident from (5) and (8) that F_2 annihilates any function of a. By contrast, use of (1), (11), and (17) shows that

$$\exp(F_2)\ a^{n/2} \exp(in\phi) = \exp(i2n\pi w)\ a^{n/2} \exp(in\phi) . \tag{20}$$

Consequently, the functions $\exp(in\phi)$ are eigenfunctions of F_2 with eigenvalues $i2n\pi w$:

$$F_2 \exp(in\phi) = i2n\pi w \exp(in\phi) \ . \tag{21}$$

This result can also be obtained by direct evaluation of the Poisson bracket $[f_2, \exp(in\phi)]$.

The determination of h as given in (18) is now straightforward. Inserting (19) into (12) and making a Fourier expansion, one finds

$$f_b = \sum_{-\infty}^{\infty} c_n(a) \exp(i2n\phi) \tag{22}$$

where

$$\begin{aligned} c_n = 4\pi Da \int_0^1 \int_0^1 du\, dv\, v \exp(-3au^2v^2) \\ \times [I_n(3au^2v^2) - I_n'(3au^2v^2)] \ . \end{aligned} \tag{23}$$

Here the symbols I_n denote modified Bessel functions, and use has been made of the standard relations[5]

$$\exp(x \cos y) = \sum_{-\infty}^{\infty} I_n(x) \exp(iny) \tag{24a}$$

$$I_{n+1} + I_{n-1} = 2I_n' \ . \tag{24b}$$

Now insert (22) into (18) and use the eigenfunction property (21). The result is that h is given in complex form by the expression,

$$h = -2\pi wa + \sum_{-\infty}^{\infty} c_n(a)[2n\pi w/\sin(2n\pi w)]\exp[2in(\phi + \pi w)] \ , \tag{25a}$$

and in real form by the expression,

$$h = -2\pi wa + c_o(a) + 2\sum_{1}^{\infty} c_n(a)[2n\pi w/\sin(2n\pi w)]\cos[2n(\phi + \pi w)] . \tag{25b}$$

The expressions (25) provide a generalization of the Courant-Snyder invariant through first order in the beam-beam interaction strength. Upon inspecting them, several points are immediately evident:

(a) Resonances occur and the formulas diverge whenever the tune w is of the form

$$w = k/(2N) \tag{26}$$

where k and N are integers. Thus there are resonances at half-integer tunes, quarter-integer tunes, sixth-integer tunes, etc. This was to be expected because u(q) as given by (3) contains no even powers and all odd powers of q.

(b) The strengths of the various order resonances are proportional to $nc_n(a)$. Using the large n expansion,[5]

$$I_n(x) \sim \exp\,[-\,n\,\log(2n/ex)]\ , \tag{27}$$

one finds from (23) that the strengths of the various resonances fall off faster than exponentially as n is increased. Therefore the sizes of various resonance features in phase space should decrease rapidly with the order of the resonance. Moreover, these features should decrease in size according to their proximity to the origin in phase space.

It also follows that (25) converges rapidly at all tune values for which w is badly approximated by rationals. Indeed, the points in tune space where (25) fails to converge are of measure zero.[6]

(c) With the aid of time reversal invariance it can be shown from (16) that the locations of various features in phase space such as fixed points and separatrices must be symmetric about the line $\phi = \pi/2 - \pi w$ for all beam-beam interaction strengths. (Note that according to (19), the line $\phi = 0$ corresponds to the p axis.) Because f_b as given by (12) is even in z_1, there is also symmetry in phase space with respect to inversion through the origin. Examination of (25) shows that both of these symmetries are present in h.

To calculate the behavior of $\mathcal{M}$ exactly at and near resonance, it is necessary to work with powers of $\mathcal{M}$. For example, consider m'th order resonances. Then m = 2N and tunes near an m'th order resonance value can be written in the form

$$w = k/m + \delta \tag{28}$$

where δ measures departure from exact resonance. Moreover, it can be shown that there is a Lie operator H_r such that $\mathcal{M}^m$ can be written in the exponential form $\exp(mH_r)$ at and near resonance without divergence difficulties Finally, there is again an effective Hamiltonian h_r corresponding to H_r that is given in this case by the formula

$$h_r = (\delta/w)f_2 + (\delta/w)F_2 \left\{1 - \exp[-m(\delta/w)F_2]\right\}^{-1} \tag{29}$$
$$\times \left\{1 + \exp[-F_2] + \exp[-2F_2] + \ldots + \exp[-(m-1)F_2]\right\} f_b + \ldots .$$

Upon inserting (22) into (29), one finds

$$h_r = -2\pi\delta a + c_o(a) + 2\sum_1^\infty c_n(a)[2n\pi\delta/\sin(2n\pi w)]\cos[2n(\phi + \pi w)]. \tag{30}$$

It is evident that the expression for h_r is well behaved near by and exactly at the resonance value $\delta = 0$.

As a specific example, consider the case of fourth order resonances. Near a one-quarter tune $k = 1$, $N = 2$, and

$$w = 1/4 + \delta \ . \tag{31}$$

Thus one finds for small δ that

$$\begin{aligned} 2n\pi\delta/\sin 2n\pi w &= 2n\pi\delta/\sin(n\pi/2 + 2n\pi\delta) \\ &= O(\delta) \text{ for n odd} \\ &= (-1)^{n/2} + O(\delta^2) \text{ for n even.} \end{aligned} \tag{32}$$

Consequently, neglecting terms of order δD and δ^2, one has in this case for h_r the expression

$$h_r = -2\pi\delta a + c_o(a) + 2 \sum_{n \text{ even}} (-1)^{n/2} c_n(a) \cos[2n(\phi + \pi w)] . \tag{33}$$

Because $-h_r$ acts as an effective Hamiltonian, the fixed points of M^4 are the equilibrium points of h_r. These points are therefore the solutions to the equations

$$0 = \partial h_r/\partial a = -2\pi\delta + c_o'(a) - 2c_2'(a)\cos 4(\phi + \pi w) + \ldots \tag{34a}$$

$$0 = \partial h_r/\partial\phi = 8c_2(a)\sin 4(\phi + \pi w)$$
$$- 16c_4(a)\sin 8(\phi + \pi w) + \ldots . \tag{34b}$$

The solutions to (34b) are readily found to be

$$\phi + \pi w = 0,\ \pi/4,\ 2\pi/4,\ \ldots,\ 7\pi/4 . \tag{35a}$$

When these solutions are inserted into the "radial" equation (34a), it takes the simple form

$$0 = -2\pi\delta + c_o'(a) \pm 2c_2'(a) + 2c_4'(a) + \ldots . \tag{35b}$$

Thus, as expected, there are 8 fourth-order fixed points when the tune is near a quarter.

The nature of these fixed points can be obtained by expanding h_r about them. At the fixed points one finds the results

$$\partial^2 h_r/\partial a^2 = c_o''(a) \pm 2c_2''(a) + 2c_4''(a) \ \ldots \tag{36}$$

$$\partial^2 h_r/\partial a\partial\phi = 0$$

$$\partial^2 h_r/\partial\phi^2 = \pm 32 c_2(a) - 128\ c_4(a) + \ldots .$$

It follows that if the equations (36) are dominated by their first terms, then the fixed points are alternately elliptic and hyperbolic (stable and unstable), as also expected,(7) because the quadratic form corresponding to (36) is either definite or mixed.

Note, moreover, that equations (36) and all the higher order terms in the Taylor series expansion about a fixed point are linear in the beam-beam interaction strength. Consequently, for small beam-beam interaction strength, the size and shape of resonant islands and their associated separatrix structure are <u>independent</u> of the beam-beam interaction strength, and are dependent <u>only</u> on their location in phase space. Only the width of the resonance in tune space, i.e., the rate at which various features move as δ is changed, depends on the interaction strength. This latter dependence can be inferred from (34a) and (35b).

3. Numerical Results

A proper study of the usefulness of h and h_r involves the numerical integration of the trajectories that they generate, or at least a determination of their level lines, and a comparison of these results with points generated by iterating M and M^m numerically. Such a comparison has been made in a similar but simpler problem involving the insertion of a short sextupole element into a ring.(2) In that case the quantitative agreement proved to be excellent, and similar agreement is expected for this problem as well.

However, because of the complexity of evaluating the coefficients $c_n(a)$, the equivalent study has not yet been carried out for the present problem. Instead, a preliminary exploration of the nature of M has been made by studying the points obtained by iterating M numerically. In this section it will be shown that M indeed does have all the qualitative properties that were predicted in the previous section.

Figures 2 through 5 show phase-space plots generated by successive iterates of M for various initial conditions and tune values. The phase-space coordinates range over (-2, 2), and the scale is chosen so that the beam lies within (-1, 1).[1] The tunes are near the resonant values 1/2, 1/4, 1/6, and 1/8 respectively, and the beam-beam interaction strength is 10^{-2}. Observe that the size of resonance features, e.g., island dimensions, indeed do decrease with increasing order of the resonance. Symmetry about the line $\phi = \pi/2 - \pi w$ and inversion symmetry are evident. The number and nature of fixed points is as anticipated.

Figure 6, which appears to be almost identical to Fig. 3, was obtained by running at a tune $w = 0.253$ and a beam-beam interaction strength of 5×10^{-3}. It shows, as predicted, that the size of resonant features is independent of the size of the interaction strength provided the tune is suitably adjusted so as to make the features appear in the same region of phase space. Note that according to (35b), when the size of the c_n is halved, δ should also be halved to keep the radial location of fixed points the same. Examination of the tune values for Figs. 3 and 6 shows that this is indeed the case. When the tune is thus adjusted, there is a slight change in the angular location of the fixed points in accord with (35a).

Figure 7 shows a tenth-order resonance obtained by running near a tune of 1/10. It was not shown as part of the sequence of Figs. 2 through 5 because the island structure becomes too small to see when (by adjusting the tune) it is located closer to the origin. This example verifies that the sizes of resonant features decrease with proximity to the origin, and in fact the higher the order of the resonance, the more rapid is the decrease.

Figure 8 shows the result of running with a nonresonant tune of 77/100. On the scale shown and for the number of iterations made, there seems to be no evidence that any points will leave the beam envelope. The nature of the map and any tendency for points to move off what appear to be invariant curves could be examined in finer detail by studying the value of h(z) at each point. Because h generalizes the Courant-Snyder invariant, its variations could be used as a kind of "magnifying glass" to give evidence for small scale homoclinic or stochastic behavior that is not otherwise discernible to the naked eye and that might lead to eventual particle loss. This method has been used to show that particle motion in the Van Allen radiation belts is not integrable.[8]

Figure 9 illustrates that stochastic behavior indeed can occur for the beam-beam problem if the interaction strength is large enough and the tune (taking into account its depression by the beam-beam interaction) is sufficiently close to a resonant value. The stochastic behavior in this case leads to particle losses within a few hundred turns.

4. Concluding Remarks and Comments

Operation of ISABELLE with each 1/6 lattice section having a tune near a multiple of 1/2, 1/4, 1/6, or 1/8 corresponds to operating the total ring near an integer, half integer, or quarter integer tune. Because operation of the total ring near any of these tunes is probably already precluded by structure resonances in the ring, the first beam-beam interaction resonance of significance is at least of tenth order. Figure 7 illustrates that the tenth-order resonance structure is small even when it is far from the phase-space origin, and consequently it is even smaller when it is within the beam. This observation, and the regular behavior found in the nonresonant case of Fig. 8, give preliminary evidence that within the model employed, the beam-beam interaction at its contemplated strengths should not lead to particle loss. However, in accord with our earlier comment, it would be worthwhile to examine the behavior of h(z) and $h_r(z)$ for evidence of small scale stochastic behavior.

The conclusion that resonances below tenth order are not significant depends on the assumption that all 6 interaction regions and all 6 lattice periods are identical. The validity of this assumption should be examined, and the effect of lower order resonances should be re-examined when the 6 interaction regions are all slightly different.

Finally, consideration should be given to the possible effect of adding nonlinearities to the transfer map for the ring. It is anticipated that the addition of suitable nonlinearities, perhaps by the use of octupoles, would lead to a reduction in the size of beam-beam interaction resonance structures. In particular, it would then no longer be the case that the size of resonant structures would depend only on their location in phase space and not on the interaction strength. It might turn out, of course, that the ring nonlinearities required to achieve a significant effect would be difficult to obtain or would be undesirable for other reasons.

Acknowledgment

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Figure Captions

1. Schematic representation of particle motion in a storage ring and a colliding beam region.

2. Phase-space plot generated by succesive iterations of the transfer map $\mathcal{M}$ for various initial conditions. The tune is near one half. The coordinates extend from - 2 to 2, and are normalized in such a way that the beam will be within the unit circle under actual operating conditions. The beam-beam interaction strength is 10^{-2}.

3. Phase-space plot when the tune is near one fourth.

4. Phase-space plot when the tune is near one sixth.

5. Phase-space plot when the tune is near one eighth.

6. Phase-space plot when the interaction strength is half that of Fig. 3. The initial p,q values are the same as in Fig. 3, and the tune is adjusted to make various phase-space features match those in Fig. 3.

7. Phase-space plot near a tune of one tenth.

8. Phase-space plot for a nonresonant tune.

9. Phase-space plot showing stochastic behavior for large beam-beam interaction strength. The reader is invited to draw the symmetry line $\phi = \pi/2 - \pi w$.

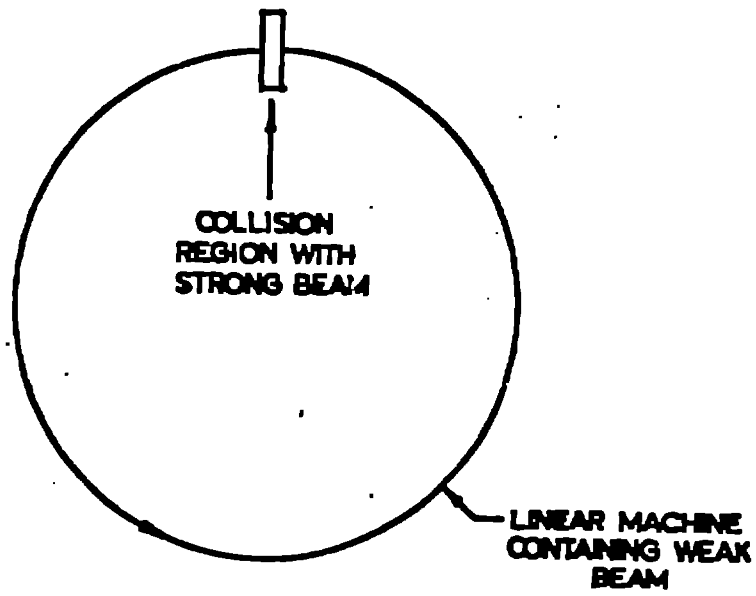
COLLISION
REGION WITH
STRONG BEAM
LINEAR MACHINE
CONTAINING WEAK
BEAM

Fig. 1

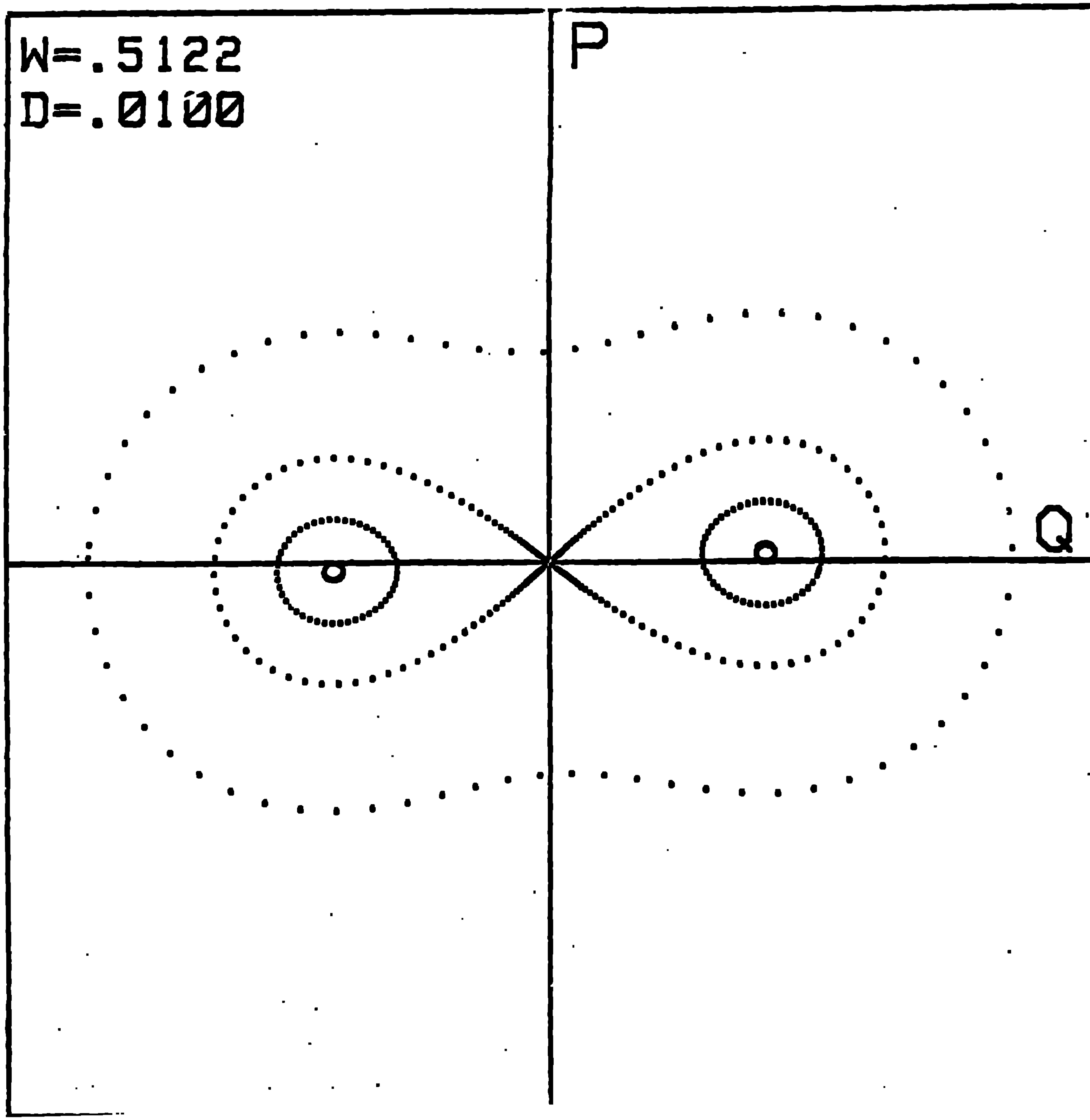

Fig. 2

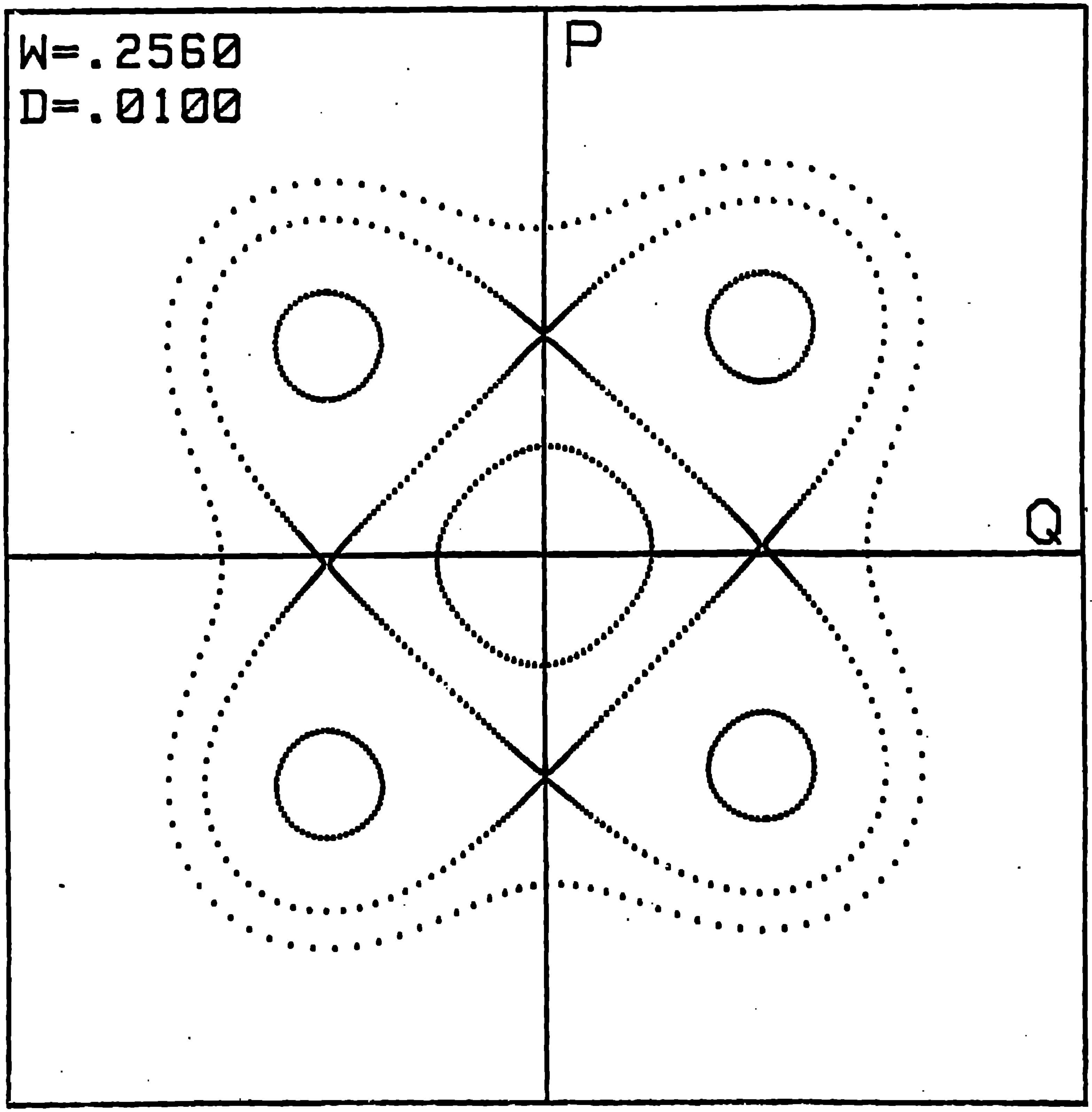

Fig. 3

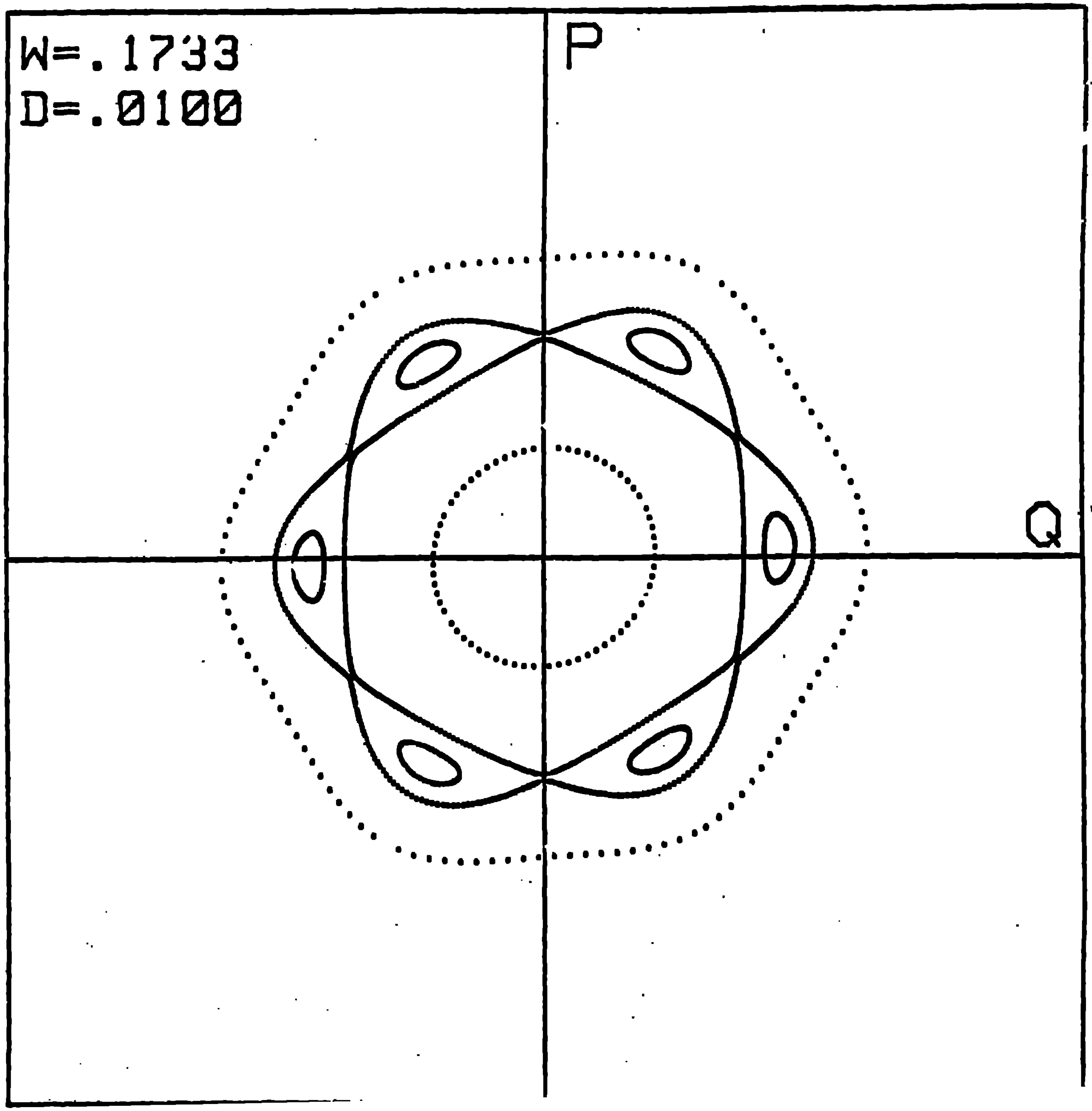

Fig. 4

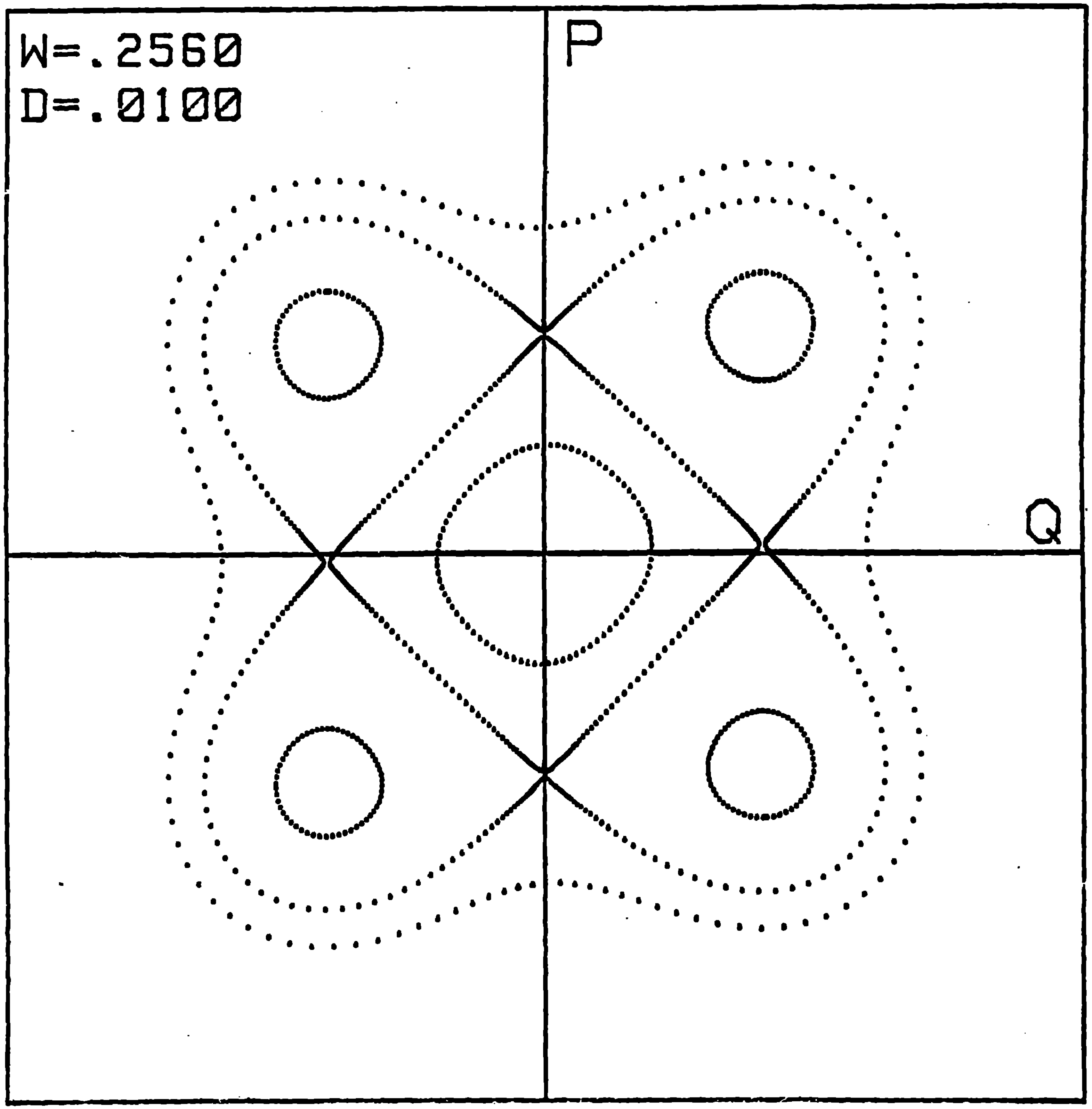

Fig. 3

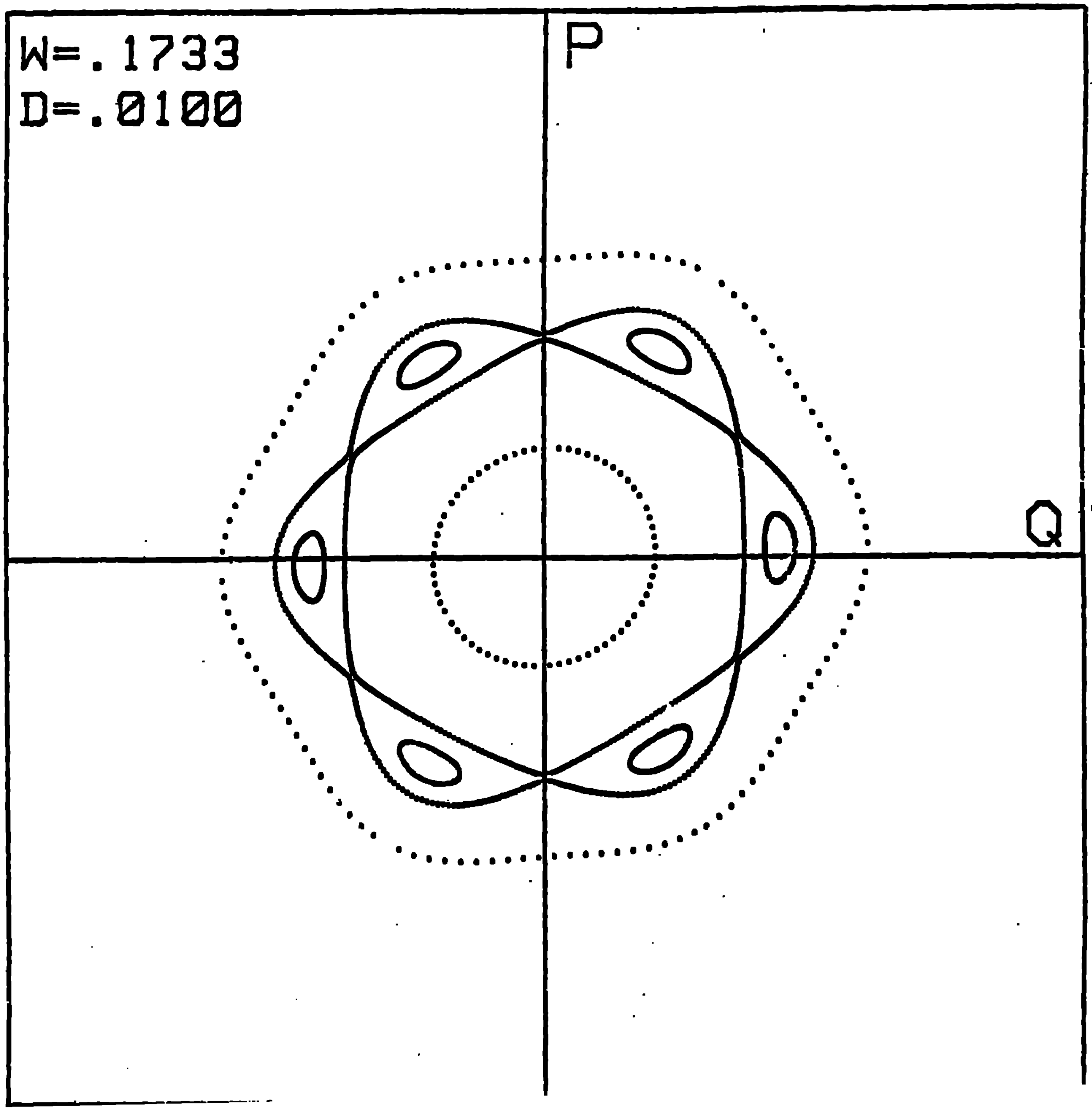

Fig. 4

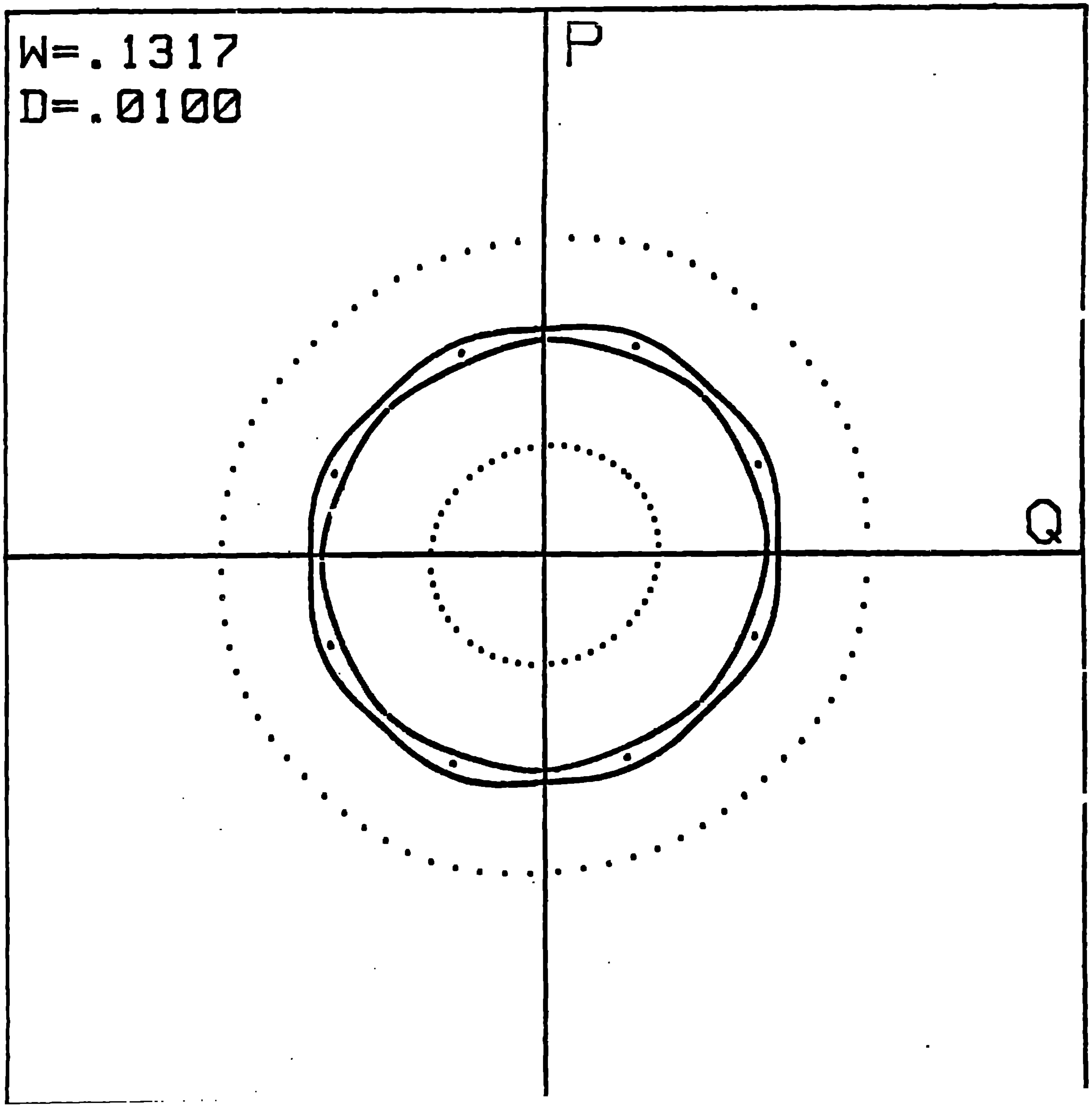

Fig. 5

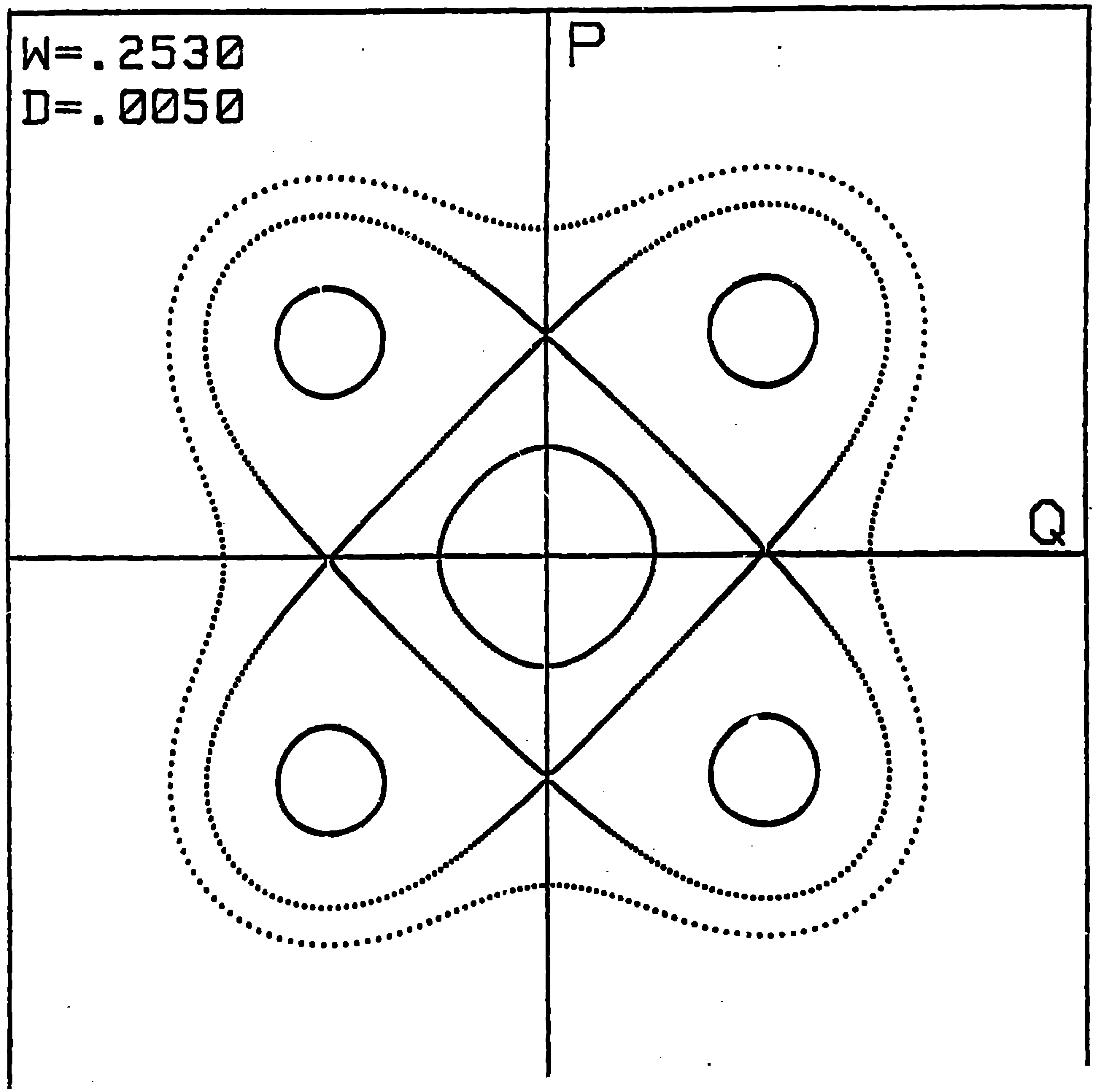

Fig. 6

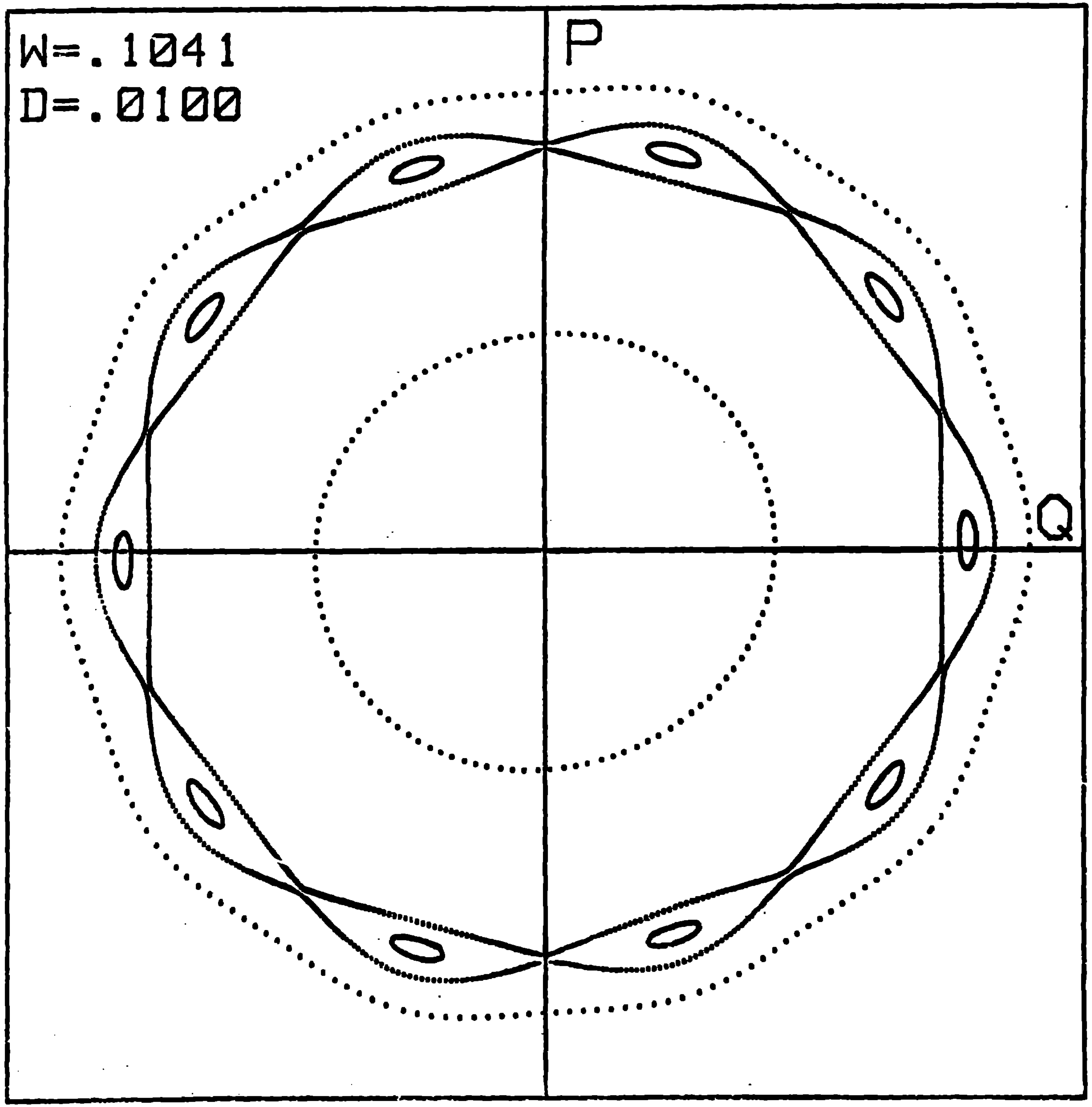

Fig. 7

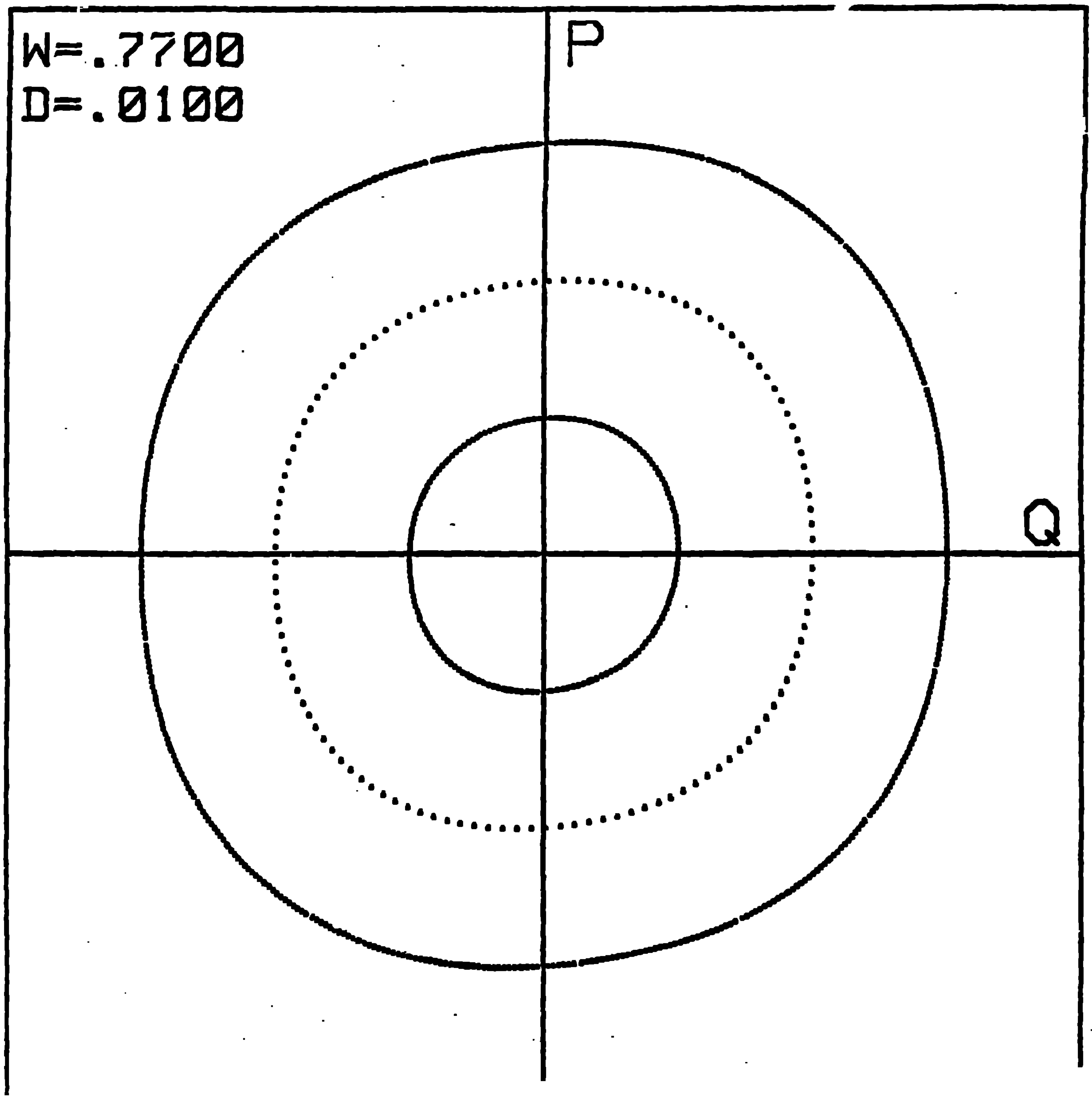

Fig. 8

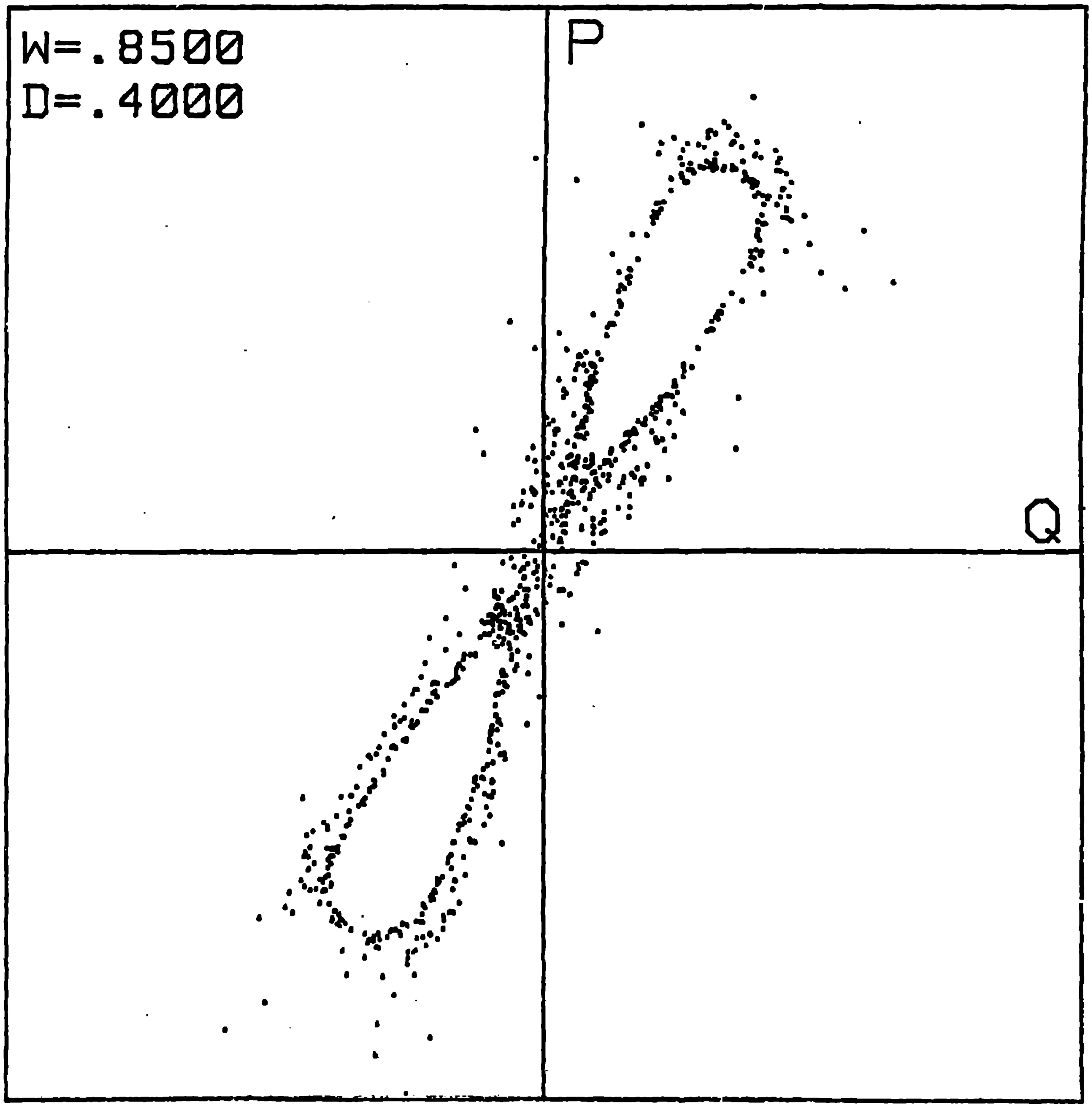

Fig. 9